Mechanics :- It is the branch of physics which deals with the study of effect of force system acting on a particle (or) a rigid body which may be at rest (or) in motion. The purpouse of Mechanics is to explain and predict physical Phenomena and thus to lay the foundation for engineering application.

Idealization in Mechanics

1. Rigid body
2. Concentrated force (point load)

Rigid body : A Rigid body is one in which all particles remans at fixed distances from each other irrespective of forces that act on the body. It does not deform under the action of external forces.

Laws of Mechanics

1. Newton's first law of motion.

Every body continues in its state of rest (or) of uniform motion in a straight line unless an external unbalanced force acts on it. It tells about principle of the equilibrium of forces.

2. Newton's second law of motion.

The rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of applied force.

$$F = \frac{mv - mu}{t} = \frac{m(v-u)}{t}$$

$$\therefore F = ma.$$

This law forms the basis for most of the analysis in dynamics.

3. Newton's Third Law of Motion:

To every action, there is an equal and opposite reaction.

Concept of force

Force is a useful and simple way of describing a very complex physical interactions between bodies.

Effect of force

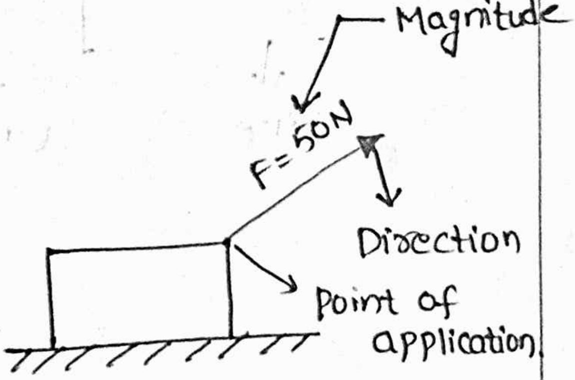
The force changes (or) tends to change the motion of the body acted on.

Characteristics of Force

a) Magnitude

b) Direction

c) point of application.



UNIT OF FORCE

It is a force required to produce an acceleration of $1 m/s^2$ in a body of mass 1kg.

force = Mass × acceleration

$F = m \times a$.

$1N = 1kg \times 1m/s^2$.

Unit :- Newton (N).

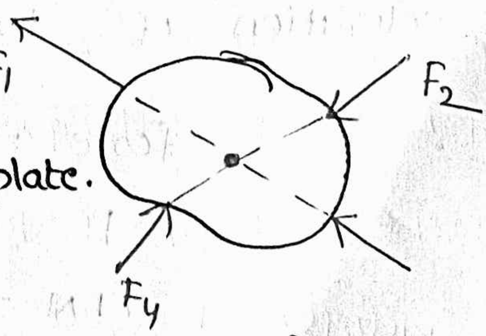
Classification of Force System

Force system.

- Coplanar
  - Concurrent
  - Parallel
    - Like
    - Unlike
  - Non-concurrent
- Non-coplanar
  - Concurrent
  - Parallel
    - Like
    - Unlike
  - Non-concurrent

Concurrent Force System

The Line of action of all forces meet at a common point. is called Concurrent force system.



Ex :- Members of a truss connected to a Gusset plate.

Non-concurrent

The Line of action of all forces should not meet at a common point.

Co-planar force system :-

If Line of action of all the forces Lies in the same plane is called co-planar force system.

Non-Coplanar force system :-

*

If the Line of action of all the forces ~~k~~ not lie in the same plane is called Non-coplanar force system.

~~CoxP~~ Collinear force system.

*

If Line of action of all the forces Lie on the same Line is called Collinear force system.

Non-Collinear force system.

*

If Line of action of all the forces not Lie on the same Line is called non-Collinear system of forces.

parallel force system :-

If the Line of action of all the forces in the system are parallel to each other then it is called a parallel force system.

(i) Like parallel forces

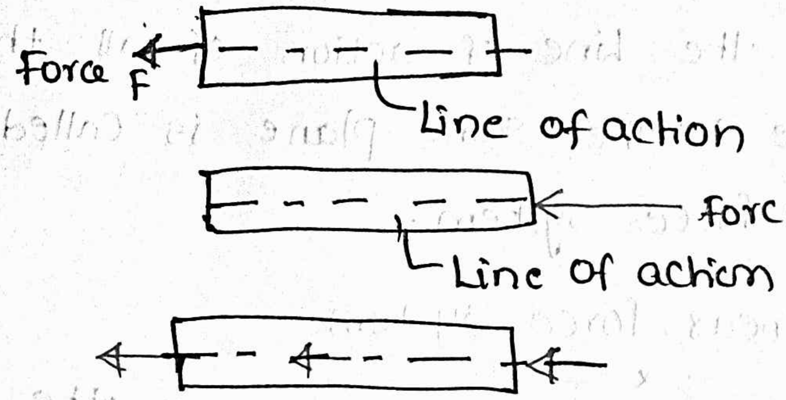
↑↑↑↑

(ii) unlike parallel forces

↓↑↓↑

Transmissibility of force

The principle of transmissibility states that the ~~eff~~ external effects of the force are independent of the point of application of the force along Its line of action.



Note: If we more the force along the line action (any where in the Line) the effect is same.

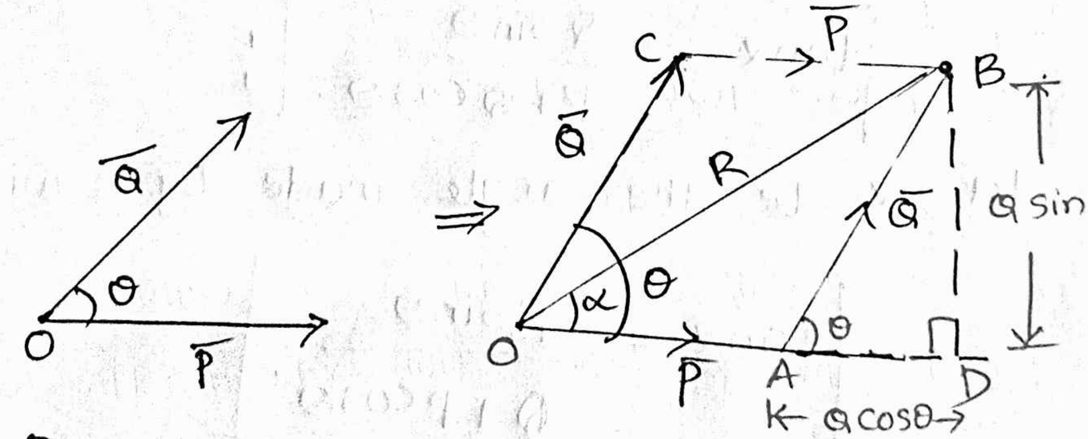
Composition of forces

Forces may be combined (added) to obtain a single force which produces the same effect as the original system of forces. This single force is known as resultant force. The process of finding the resultant of forces is called composition of forces.

Force is a vector quantity. The method of addition of addition of forces (vectors) is based on the parallelogram law.

Parallelogram Law of Forces

The resultant of any two non-collinear concurrent forces may be found by this law which states that " If two forces acting simultaneously on a body at a point are represented in magnitude & direction by two adjacent sides of a parallelogram, then their resultant is represented in magnitude & direction by the diagonal of parallelogram which passes through the point of intersec-tion of two forces.



θ = angle between two forces

α = angle between Resultant and one of the two forces

proof :

Draw a perpendicular from point B on OA extend-ed, meeting at point D. As OC is parallel to AB and OC = AB = Q , OA = P and OB = R

In ΔODB

$$OB^2 = OD^2 + BD^2$$

$$OB^2 = (OA + AD)^2 + BD^2$$

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$R^2 = P^2 + 2PQ\cos^2\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

Magnitude of Resultant $\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$

from ΔOBD, let α be the angle made by R with P,

$$\tan\alpha = \frac{BD}{OD} = \frac{BD}{OA + AB}$$

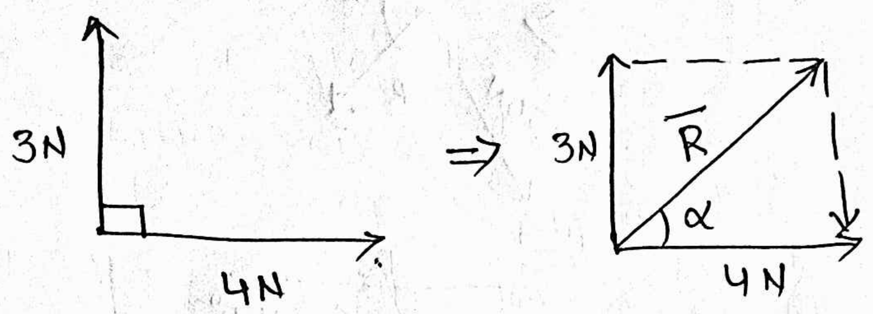
$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}};$$

Let α be the angle made by R with Q,

$$\boxed{\tan\alpha = \frac{P\sin\theta}{Q + P\cos\theta}}$$

Problems on parallelogram Law [Composition of forces]

1) Find the Resultant of given forces.



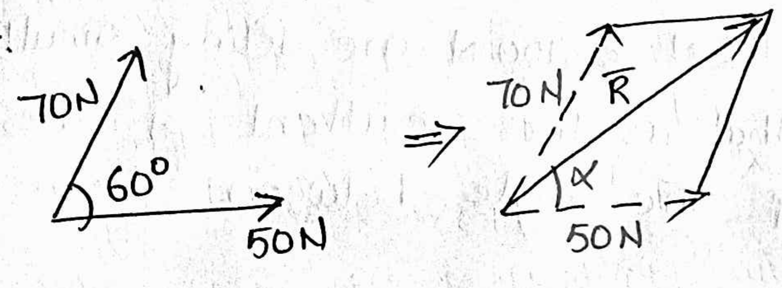
$$\therefore R = \sqrt{4^2 + 3^2 + 2 \times 3 \times 4 \cos 90^\circ}$$

$$R = 5N$$

$$\tan\alpha = \frac{3 \sin 90^\circ}{4 + 3 \cos 90^\circ}$$

$$\therefore \alpha = 36.87^\circ$$

Ex:2



$$R = \sqrt{50^2 + 70^2 + 2 \times 50 \times 70 \cos 60^\circ}$$

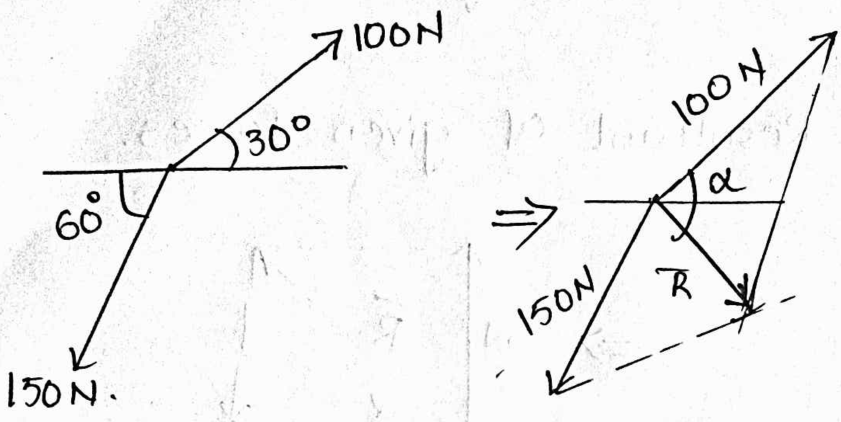
$$R = 104.4 \text{ N.}$$

$$\tan\alpha = \frac{70 \sin 60^\circ}{50 + 70 \cos 60^\circ}$$

$$\tan\alpha = 0.7132$$

$$\alpha = 35.5^\circ$$

Ex:3 Find Resultant & Its direction.



$$R = \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \cos 150^\circ}$$

$$R = 80.74\ N.$$

$$\tan\alpha = \frac{150 \sin 150^\circ}{100 + 150 \cos 150^\circ}$$

$$\tan\alpha = |-2.51|$$

$$\alpha = 68.26^\circ.$$

Ex

Two forces of 100 N & 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?

Ex

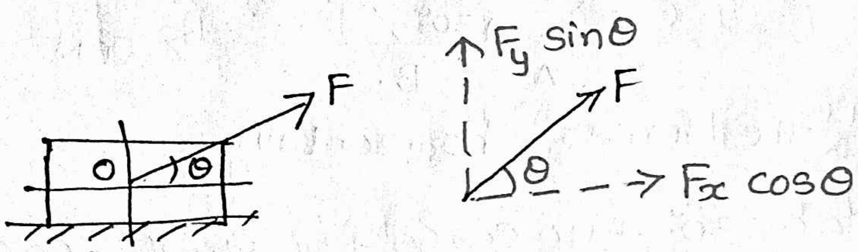
Two forces act at an angle of 120°. The bigger force is 40N and the resultant is perpendicular to the smaller one. Find smaller force.

Ex

Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But they act at 60°, their resultant is $\sqrt{13}$ N.

RESOLUTION OF A FORCE

The process of breaking the force into mutually perpendicular components which are equivalent to the given force is called Resolution of a force.

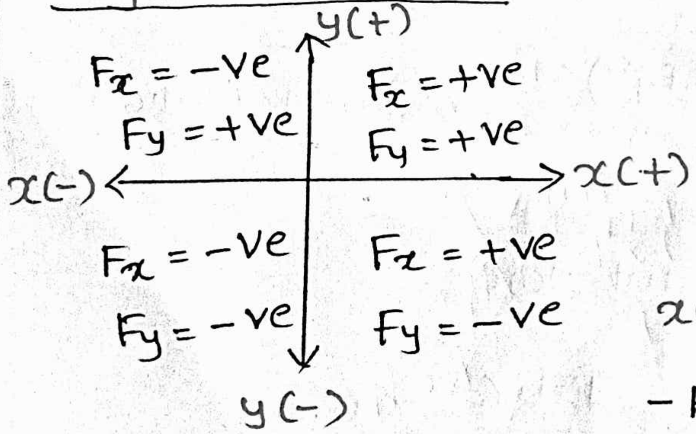


Magnitude of Resultant

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

Direction → $$\theta = \tan^{-1}\left[\frac{\Sigma F_y}{\Sigma F_x}\right]$$

Sign conventions



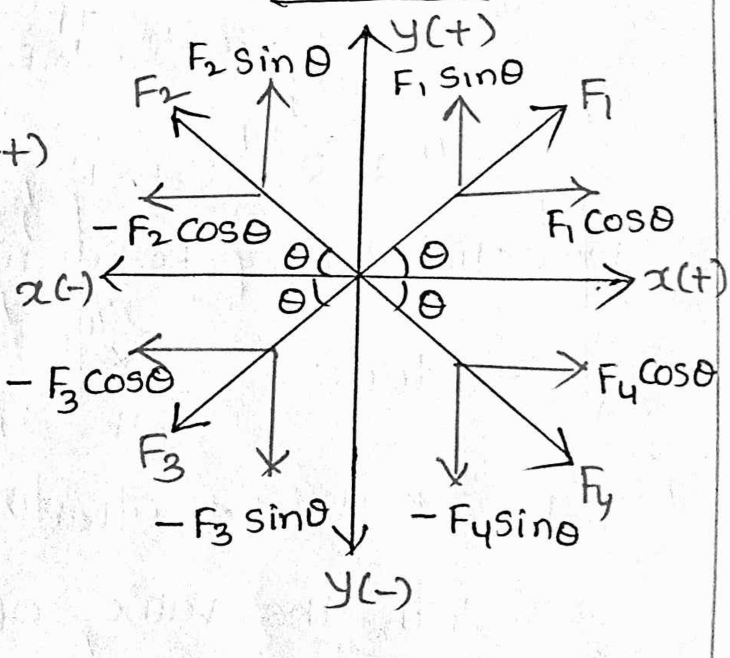
↑ Upward +ve

↓ Downward −ve

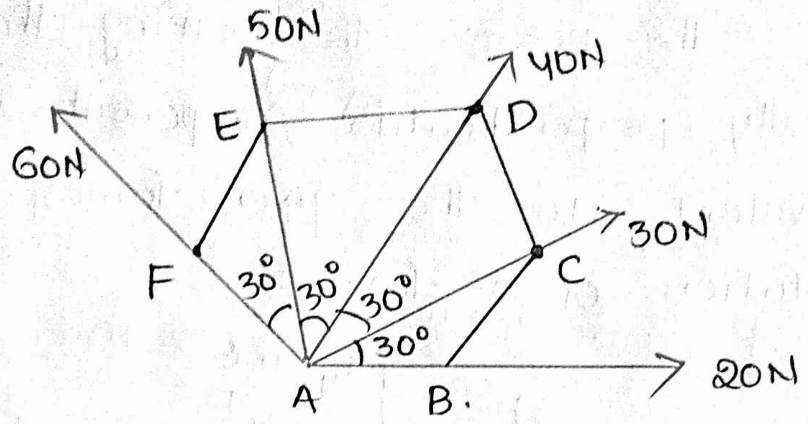
→ Right +ve

← Left −ve.

Example



Find Magnitude & direction of a Resultant Force.



Resolving all the forces horizontally.

$\Sigma F_x = 20 + 30\cos 30^\circ + 40\cos 60^\circ + 50\cos 90^\circ + 60\cos(120^\circ)$

$\Sigma F_x = 36.N$

Resolving all the forces vertically.

$\Sigma F_y = 20 + 30\sin 30^\circ + 40\sin 60^\circ + 50\sin 90^\circ + 60\sin(120^\circ)$

$\Sigma F_y = 151.6\,N.$

Magnitude of Resultant force

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y^2)} = \sqrt{(36)^2 + (151.6)^2}$

$R = 155.8\,N$

Direction of Resultant force.

$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36} = 4.211$

$\theta = \tan^{-1}(4.211) = 76.6^\circ.$

Since both the values of ΣF_x & ΣF_y are positive therefore actual angle of resultant force lie b/w 0° to 90°

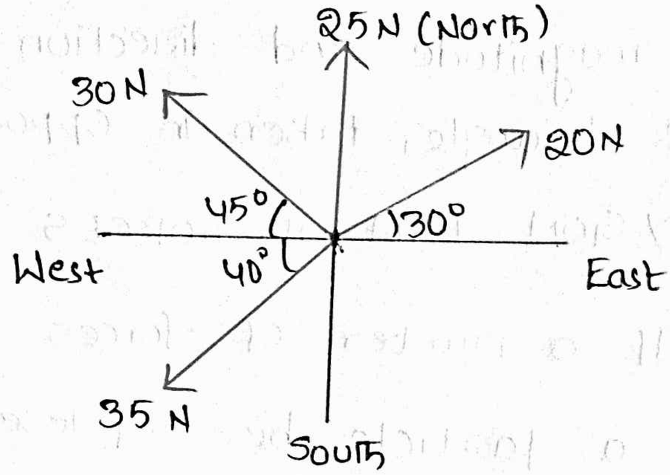
The following forces act at a point.

i) 20 N inclined at 30° towards North of East.

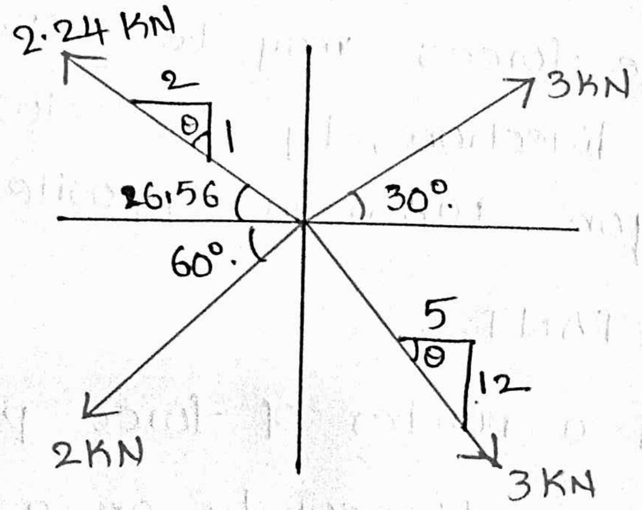
ii) 25 N towards North

iii) 30 N towards North West

iv) 30 N inclined at 40° towards South of West.



Find Resultant & Direction.



$\tan\theta_2 = 2/1$

$\theta = \tan^{-1}(2/1)$

$\theta = 63.43$

$\theta = 90 - 63.43$

$\theta_2 = 26.56$.

$\tan\theta_4 = 12/5 \Rightarrow \theta = 67.38$.

TRIANGLE LAW OF FORCES

It states, If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.

POLYGON LAW OF FORCES

If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order.

RESULTANT :

If a number of force P, Q, R... etc, are acting simultaneously on a particle, to find a single force which could replace them. i,e, which would produce the same effect as produced by all the given forces. This single force is known as Resultant.

Equilibrium :- Any System of forces which keeps the body at rest is said to be equilibrium.

Equilibrium Equations (or) Conditions

$$\Sigma F_x = 0$$

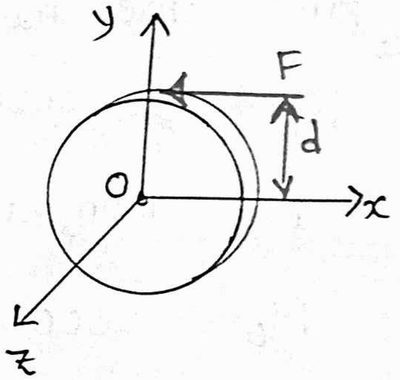
$$\Sigma F_y = 0$$

$$\Sigma M = 0.$$

Equilibriant :- The force which brings the set of forces in equilibrium is called an equilibriant.

Moment of Force

The rotational effect produced by a force is called moment of a force.



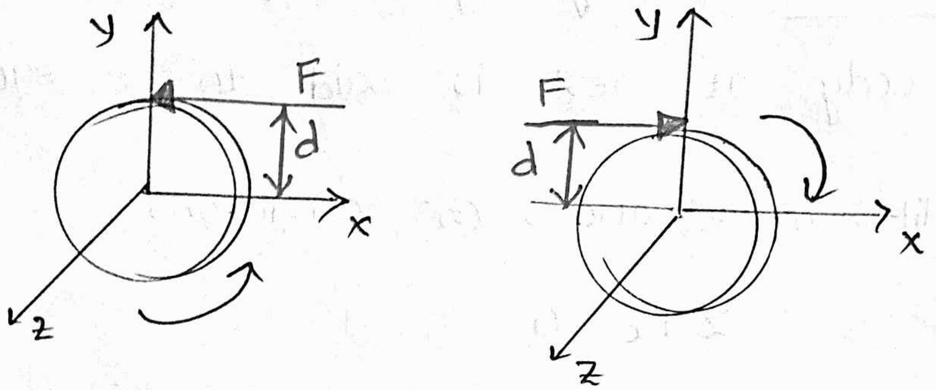
The moment of the force about an axis through O is M_O and is given as the product of magnitude of the forces 'F' and perpendicular distance 'd' from 'O' to the line of action of the force F.

$$M_O = F \times d.$$

Units : N-m.

Note: Force produces zero moment about reference Point which intersect the Line of action of force.

Sign conventions



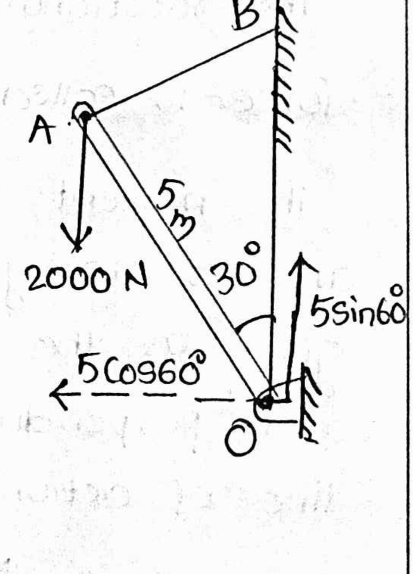
Anti Clock wise (↺) rotation Positive

Clockwise (↻) rotation Negative.

Example :-

While opening & closing of a door. Tightinging of nut by a spanner produces moment of a force.

Find the moment of force 2000N about point O shown in fig.



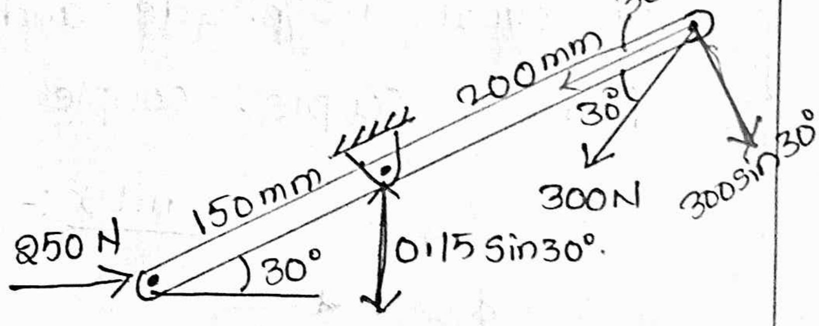
Take moment about 'O'.

$$M_O = 2000 \times 5 \cos 60^\circ$$

$$M_O = 5000 \text{ N-m} (↺)$$

Find the moment of force on lever about point 'O'

$\Sigma M_O = 0$

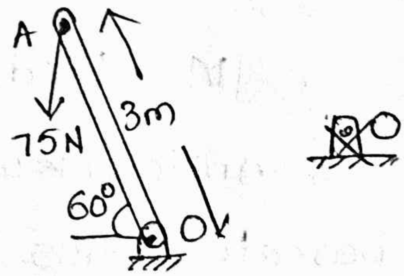


$\Sigma M_O = 250 \times 0.15 \sin 30° - 300 \times \sin 30° \times 0.35$

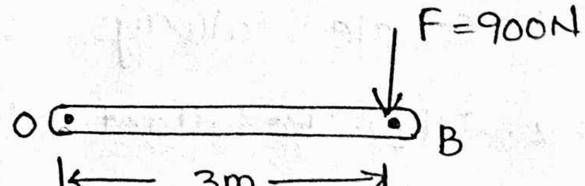
$\Sigma M_O = -33.75$ N-m

$\Sigma M_O = 33.75$ N-m (↻)

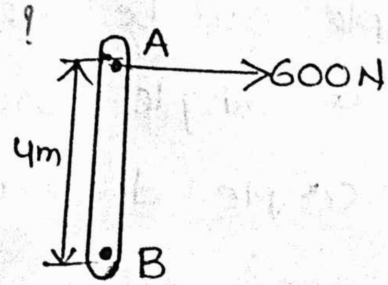
A 75 N vertical force is applied to the end of a force 3m long which is attached to a shaft at 'O'. Determine moment of force about 'O'



Find moment of a force ?

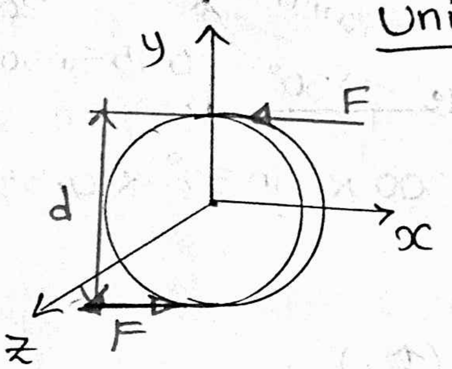


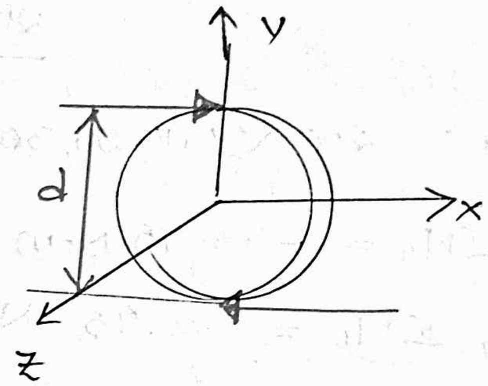
Find moment of a force !



COUPLE : Two non-collinear parallel forces of equal magnitude and in opposite direction form a couple. couple produce rotational effect.

Units :- N-m.





Moment of a Couple is the product of Common magnitude of two forces 'F' and of the perpendicular distance 'd' between the lines of action.

$M = F \times d$ (↺) Anticlockwise positive

$M = f \times d$ (↻) clock wise negative.

Charectaristics of a couple.

1. Couple always Comes with pair.
2. The Resultant of a Couple system is zero
3. Couple can be replaced by couple only not by a single force.
4. A couple does not have moment centre.

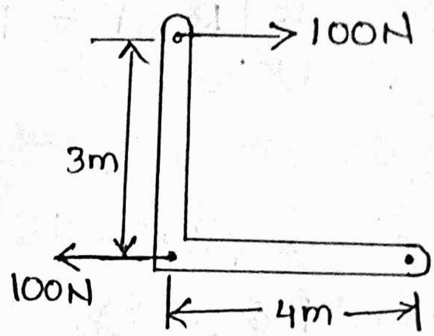
Examples for couple :-

The steering wheel of a car

Rotation of a key to lock (or) unlock

opening or closing of a tap.

Example 1 :- find moment of a Couple.



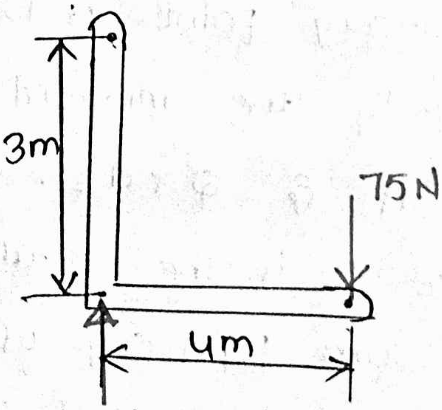
Couple M = 100×3

M = 300 N-m (↻)

Example 2 :

M = 75×4

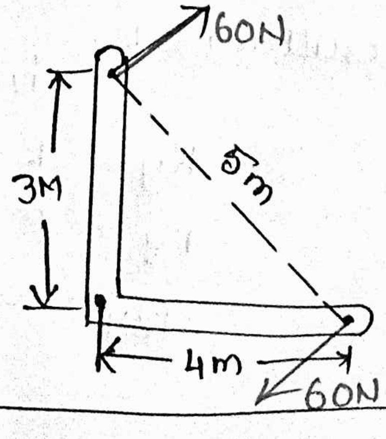
M = 300 N-m (↻)



Example 3 :-

M = 60×5

M = 300 N-m (↻)



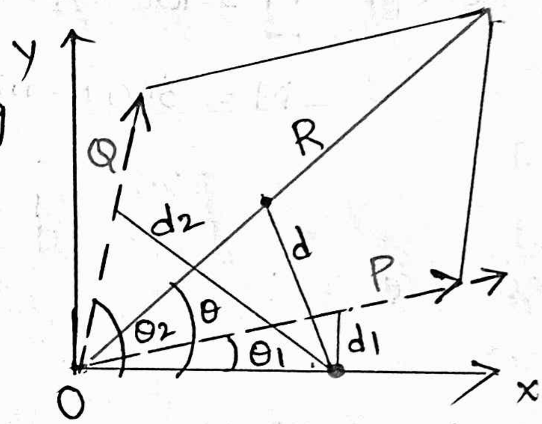
Varignon's Theorem

It States that the moment of resultant of all the forces in a plane about any point is equal to the algebric sum of moment of all the forces about the same point.

$$\boxed{R \times d = P \times d_1 + Q \times d_2}$$

proof

Consider a force 'R' acting at 'O', P & Q are the resolved components of 'R'.



The moment of 'R' about an arbitary point A is $R \times d$.

Similarly the moment of P & Q about point A are $P \times d_1$ & $Q \times d_2$. d_1 & d_2 are ⊥er distance from A.

Since R is the resultant of P & Q, it follows that the sum $P_y + Q_y$ of the y component of two forces P and Q is equal to the y-component of R_y of their Resultant R.

i, e $R_y = P_y + Q_y$

$$R_y = R \sin\theta$$
$$P_y = P \sin\theta_1$$
$$Q_y = Q \sin\theta_2$$

$\therefore R \sin\theta = P \sin\theta_1 + Q \sin\theta_2$

Multiplying both sides by length OA.

$R \times OA \sin\theta = P \times OA \sin\theta_1 + Q \times OA \sin\theta_2$

But $OA \sin\theta = d$

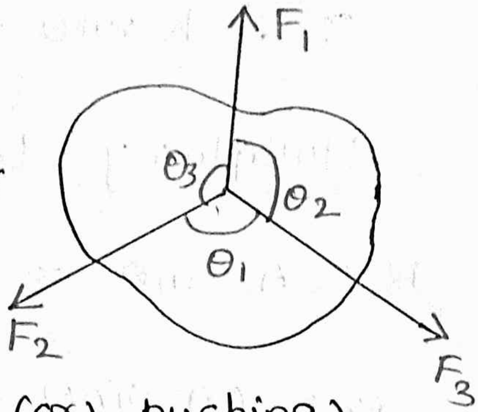
$OA \sin\theta_1 = d_1$

$OA \sin\theta_2 = d_2$

$\therefore R \times d = P \times d_1 + Q \times d_2$

Hence proved.

Lami's Theorem



If three concurrent co-planar forces acting on a body having same nature (pulling (or) pushing) are in equilibrium, then the each force is proportional to the sine of angle included between the other two forces.

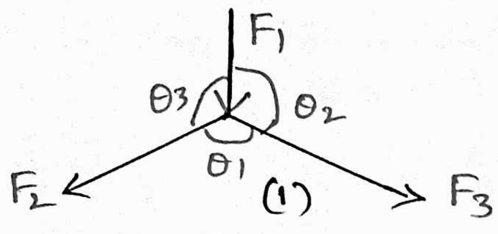
By Lami's Theorem we have

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}.$$

Limitation's of Lami's theorem.

1. It is applicable to three non-~~coplanar~~ parallel coplanar concurrent forces only.

2. Nature of three forces must be same (pushing/pulling)



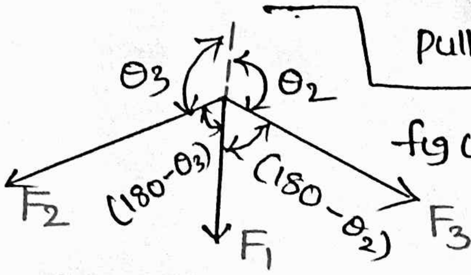
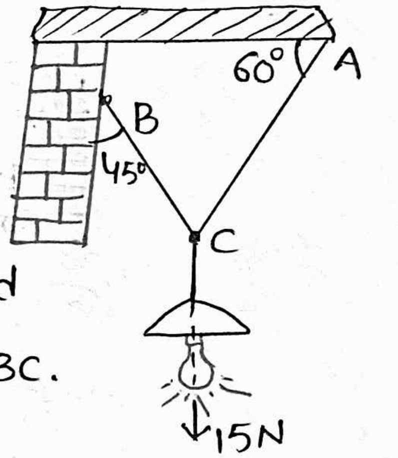


fig (2)

from the above fig(1) F_2 & F_3 forces are pulling forces but 'F_1' is pushing force. By principle of transmissibility, one can transmit F_1 on other side of point of concurrency to make three forces same nature.

Example: 1 An electric light fixture weighting 15 N hangs from a point 'c', by two strings AC & BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in fig. Determine the forces in the strings AC & BC.



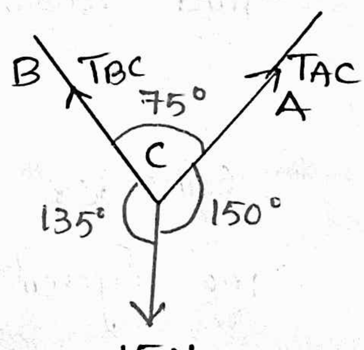
Weight at C = 15 N

T_{AC} = Tension force in string AC and

T_{BC} = Tension force in the string BC.

Applying Lami's Theorem at 'C'.

F.B.D



$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 45^\circ}$$

$$\Rightarrow T_{AC} = \frac{15 \sin 45^\circ}{\sin 75^\circ}$$

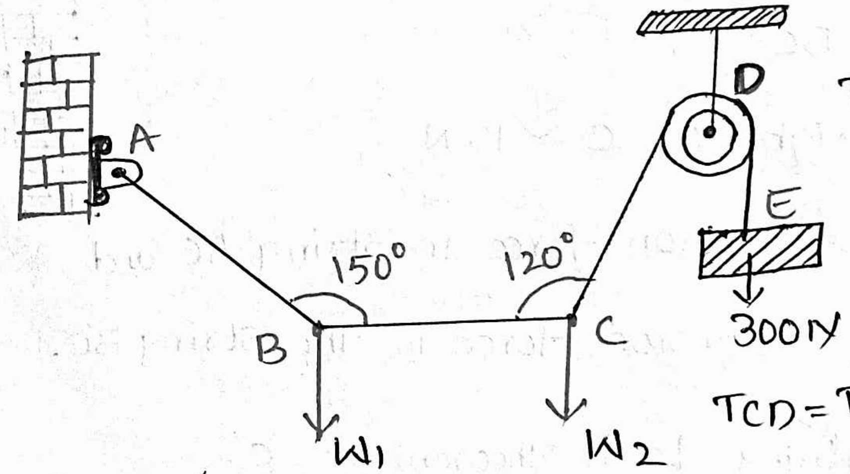
$$\therefore T_{AC} = 10.98 N$$

$$\frac{15}{\sin 75^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$T_{BC} = \frac{15}{\sin 75^\circ} \times \sin 150^\circ$$

$$T_{BC} = 7.76 N$$

Example 2 A light string ABCDE whose one end 'A' is fixed at A. has weights W1 & W2 attached to it at B and C. It passes around a small smooth peg (pully) at D carrying a weight of 300N at the free end E as shown in fig.



$T_{CD} = T_{DE}$

$T_{CD} = T_{DE} = 300N$

find Tensions in i) AB, BC & CD of the string
ii) Magnitudes of W1 & W2

Consider ABCD into two parts for sake of easy
The system of forces at joints B & C are

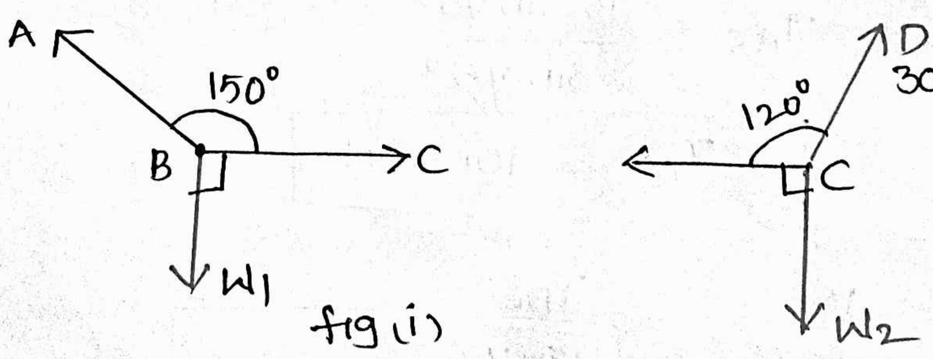


fig (i) fig (2)

from fig (2) Apply Lami's at 'c'

$$\frac{T_{BC}}{\sin(150^\circ)} = \frac{W_2}{\sin(120^\circ)} = \frac{300}{\sin 90^\circ}.$$

$$\frac{W_2}{\sin 60^\circ} = \frac{300}{1}$$

$$W_2 = 300 \times \sin 60^\circ$$

$$\boxed{W_2 = 259.8\ N}$$

$$\frac{T_{BC}}{\sin 30^\circ} = \frac{300}{1}$$

$$T_{BC} = 300 \sin 30^\circ$$

$$\boxed{T_{BC} = 150\ N}$$

Applying Lami's at B; from fig (i)

$$\frac{T_{AB}}{\sin 90^\circ} = \frac{W_1}{\sin 150^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

$$\frac{T_{AB}}{1} = \frac{W_1}{\sin 150^\circ} = \frac{150}{\sin 120^\circ} \quad \because T_{BC} = 150 N$$

$$\frac{W_1}{\sin 150^\circ} = \frac{150}{\sin 120^\circ}.$$

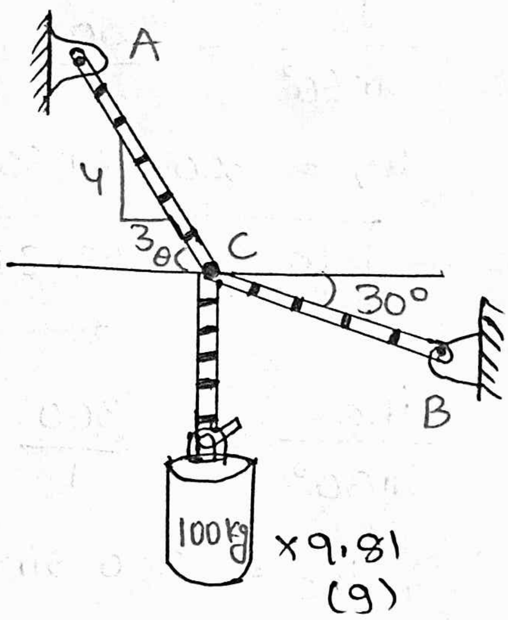
$$W_1 = \frac{150}{\sin(120^\circ)} \times \sin(150^\circ)$$

$$\boxed{W_1 = 86.6\ N}$$

$$\frac{T_{AB}}{1} = \frac{150}{\sin 120^\circ}$$

$$\boxed{T_{AB} = 173.2\ N}$$

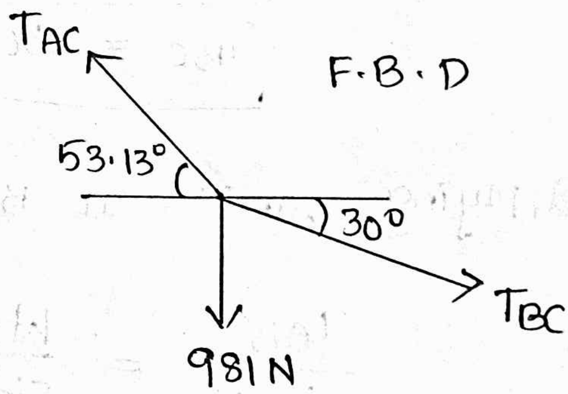
Find the Tension in each rope?



Sol

Consider the F.B.D of point "C".

By Lami's Theorem.

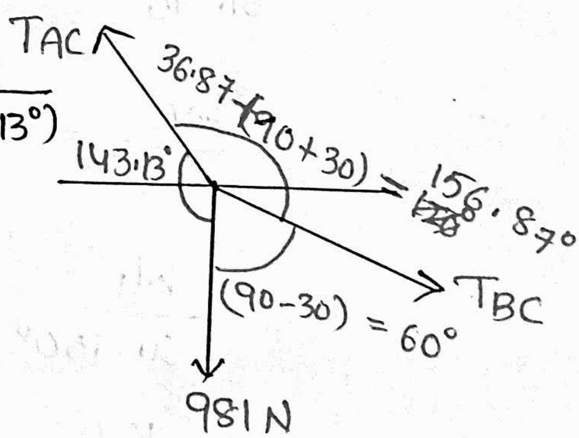


$\tan\theta = 4/3$

$\theta = \tan^{-1}(4/3)$

$\theta = 53.13°$

$$\frac{981}{\sin(156.87°)} = \frac{T_{AC}}{\sin 60°} = \frac{T_{BC}}{\sin(143.13°)}$$



$$\frac{981}{\sin(156.87°)} = \frac{T_{AC}}{\sin 60°}$$

$$T_{AC} = \frac{981 \times \sin 60°}{\sin(156.87°)}$$

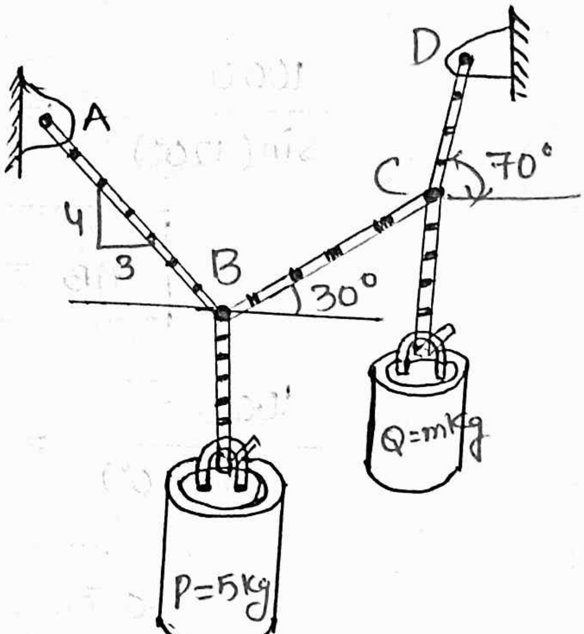
$$\boxed{T_{AC} = 2162.76\ N}$$

$$\frac{981}{\sin(156.87°)} = T_{BC}/\sin(143.13°)$$

$$T_{BC} = 981/\sin(156.87°) \times \sin(143.13°)$$

$$\boxed{T_{BC} = 1498.41\ N}$$

Block P = 5 kg and block 'Q' of mass 'm' kg is Suspended through the chord is in the equilibrium position as shown in fig. Determine the mass of block Q.

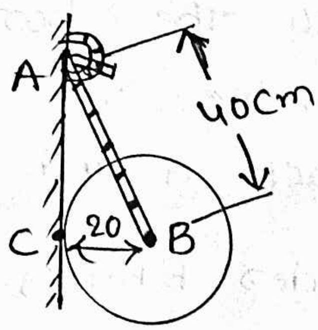


$T_{AB} = 42.79\ N$

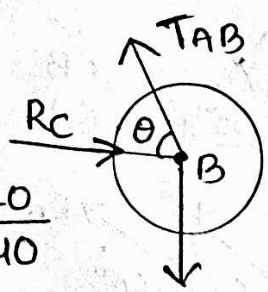
$T_{BC} = 29.64\ N$

$m = 5.678\ kg$

A circular roller of weight 100 N and radius 20 cm hang by a tie rod AB = 40 cm and rests against a smooth vertical wall at 'c' as shown in fig. Determine the tension in the rod and reaction at point C.

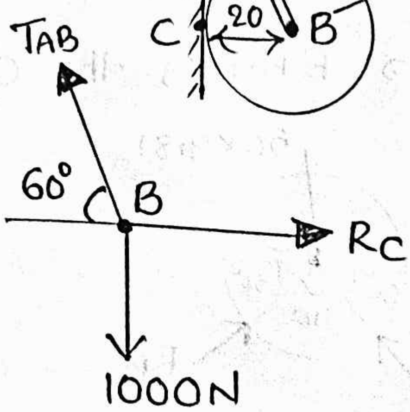


F.B.D of the roller



$\cos\theta = \frac{20}{40}$

$\theta = 60°$



Apply Lami's Equation at B.

$$\frac{1000}{\sin(120°)} = \frac{T_{AB}}{\sin 90°} = \frac{R_C}{\sin(150°)}$$

$$\frac{1000}{\sin(120°)} = \frac{T_{AB}}{\sin 90°}$$

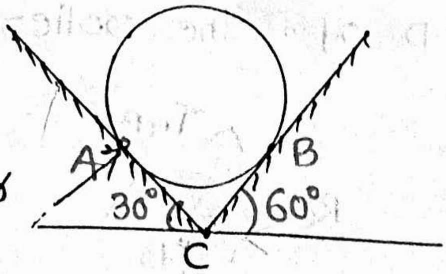
$$\boxed{T_{AB} = 1154.7\ N}$$

$$\frac{1000}{\sin(120°)} = \frac{R_C}{\sin(150°)}$$

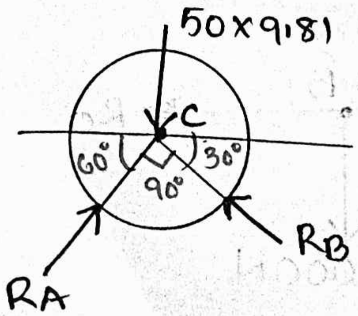
$$R_C = \frac{1000 \times \sin(150°)}{\sin(120°)}$$

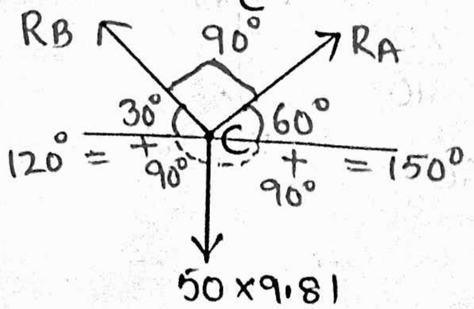
$$\boxed{R_C = 577.35\ N}$$

A cyllinder of mass 50 kg is resting on a smooth surface which are inclined at 30° and 60° to the horizontal as shown in fig. Determine the reaction at contact A and B.



Consider F.B.D of the Cyllinder





Apply Lami's Equation at 'c'

$$\frac{50\times 9.81}{\sin 90^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 150^\circ}$$

$$\frac{R_A}{\sin 120^\circ} = \frac{50\times 9.81}{\sin 90^\circ}$$

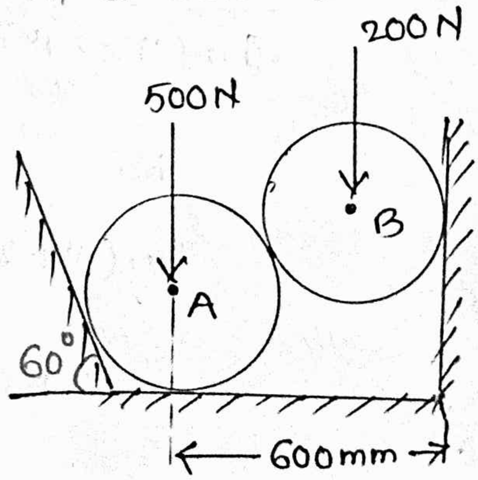
$$\boxed{R_A = 424.79\,N}$$

$$\frac{R_B}{\sin 150^\circ} = \frac{50\times 9.81}{\sin 90^\circ}$$

$$R_B = \frac{50\times 9.81}{\sin 90^\circ} \times \sin 150^\circ$$

$$\boxed{R_B = 245.25\,N}$$

Two spheres A & B resting in a smooth through as shown in fig. Radius of A & B are 250 mm & 200 mm respectively. Find Reactions?

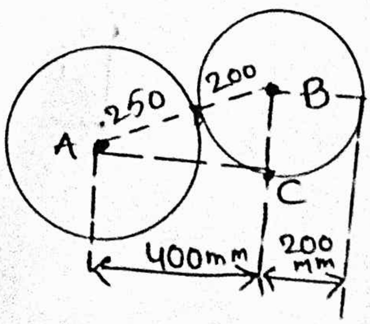


from fig $AB = 450\,mm$

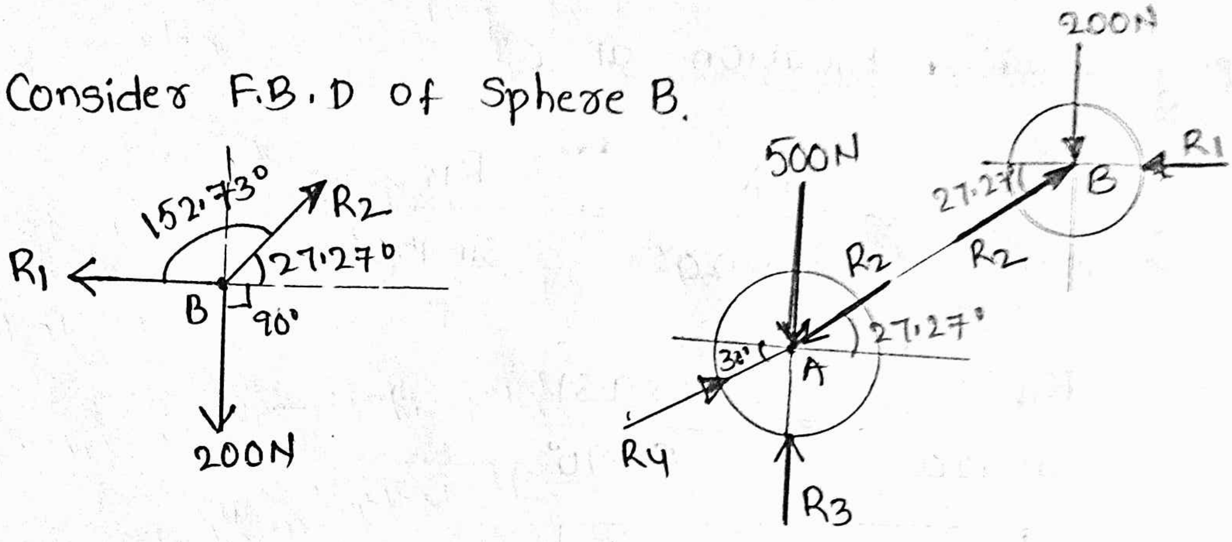
$AC = 400\,mm$

$$\cos\theta = \frac{AC}{AB} = \frac{400}{450}$$

$$\therefore \theta = 27.27^\circ$$



Consider F.B.D of Sphere B.



$$\frac{200}{\sin 152.73^\circ} = \frac{R_2}{\sin 90^\circ} = \frac{R_1}{\sin 117.27^\circ}$$

$$\frac{R_2}{\sin 90^\circ} = \frac{200}{\sin(152.73^\circ)}$$

$$\boxed{R_2 = 436.51\ N}$$

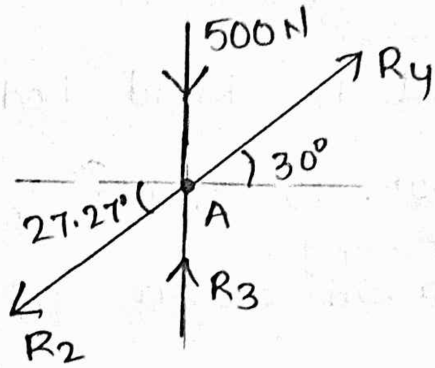
$$\frac{R_1}{\sin(117.27^\circ)} = \frac{R_2}{\sin 90^\circ}$$

$$\frac{R_1}{\sin(117.27^\circ)} = \frac{436.51}{\sin 90^\circ}$$

$$R_1 = \frac{436.51 \times \sin(117.27^\circ)}{\sin 90^\circ}$$

$$\boxed{R_1 = 388\ N}$$

Consider F.B.D of 'A'



$\Sigma F_x = 0$

$R_4 \cos 30° - R_2 \cos 27.27° = 0$

$R_4 \cos 30° - 436.51 \cos(27.27°) = 0$

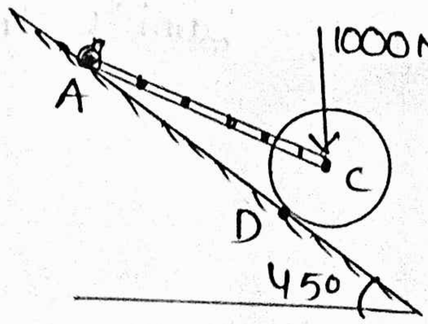
$\boxed{R_4 = 448 N}$

$\Sigma F_y = 0$

$-500 + R_3 - 436.51 \sin 27.27° + 448 \sin 30° = 0$

$\boxed{R_3 = 476 N}$

A roller of weight W = 1000 N rests on a smooth inclined plane. It is kept from rolling down the plane by string AC, as shown in fig. Find the tension in the string and reaction at point of contact D.



Important Questions

1. what is Rigid Body ?
2. what is force & Its Charectaristics ?
3. classification of System of forces ?
4. what is principle of Transmissibility ?
5. Write parallelogram law ?
6. what is triangle law & polygon's Law ?
7. What is Resultant ?
8. What is Equilibrium & Equilibriant ?
9. What is F.B.D ?
10. What is Moment of a force give Examples ?
11. What is Couple & Write practical applications ?
12. What are the charectaristics of a couple ?
13. State & prove Varignon's theorem ?
14. what is Lami's theorem ? write it's Limitations ?